

Vertical Transmission SIR Model with Vital Dynamics

Vertical transmission is the infection of offspring at birth by the mother. Examples of this type of infection might be HIV infections (vertical transmission is not the primary mode of infection for HIV and AIDS) where a fraction of births by an HIV-positive mother are infected. How does vertical transmission affect the dynamics of the infection?

Background

For diseases whose time scales are sufficiently long enough for vital dynamics to be important, they tend to be suppressed by the removal of infected and recovered by death, which are replaced by new susceptible members by birth. This can lead to endemic diseases since immunity is lost due to deaths in the recovered population. Additionally, outbreaks can be prevented if the infection rate is much smaller than the recovery and death rates. This is for diseases where new members are always susceptible to the infection but are not infected at birth. There are a number of diseases that are transmitted vertically from the mother to child during birth. Understanding the dynamics of these diseases in comparison to diseases that do not exhibit vertical transmission is important.

Some Model Requirements

- Your model will consist of the basic SIR Model which divides the population into three groups: Susceptible (S), Infected (I), and Recovered (R).
- Susceptible are infected by the Infected at a rate proportional to the number of susceptible and infected.
- The infected group recover at a rate proportional to their population size.
- The population size is constant so the number of births and deaths in the population are equal.
- Deaths occur at the same rate in each group. (You can relax this assumption to better model diseases that increase the death rate for the infected)
- Births from the recovered and susceptible groups are all always in the susceptible group but births from the infected group are split between susceptible and infected groups with a percentage becoming infected at birth.

Some Questions to Answer

- How does the SIR model with vital dynamics behave if there is no vertical transmission? Under what conditions are there an outbreak and when is the infection endemic without vertical transmission?

- With vertical transmission, how does the dynamics change? How does this compare with an infection without vertical transmission?
- How does the conditions for an outbreak change as the percent infected at birth changes?
- How does the conditions for an epidemic change as well as the endemic population size as the percent infected at birth change?
- How could this model be used to help control a disease of this type?